Towards an eigenvariety for reductive Q-groups split at p

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1. Introduction

This is all joint work with Glenn Stevens. Current methods for constructing eigenvarieties directly from cohomology with coefficients in function spaces or distribution spaces mostly work with H^0 and Banach space structures. For a general reductive $\mathbf{Q}$-group split at p, we must work with H^* for all $*$ to get at interesting automorphic representations and other phenomena. A basic technical problem arises when $* > 0$, namely that the space of boundaries is not necessarily closed in the space of cocycles. There is thus no reason to expect that H^* can carry a nice Banach structure in general.

We define an algebraic concept of "slope less than or equal to h decomposition" that behaves well under taking cohomology. We lift the U_p operators to the level of cochains, for which there is a Banach space structure that enables us to produce slope decompositions, which then descend to the cohomology. In this way we find a posteriori that the finite slope part of H^* has a nice Banach structure. We also prove a comparision theorem between cohomology with coefficients in distribution modules D and with coefficients in finite dimensional modules, if the slope is sufficiently small.

It should be emphasized that the objects we study can be computed to arbitrary precision in principle. Computations for GL(3) have been done in the ordinary case, giving proven examples of "rigidity" of certain systems of Hecke eigenvalues (joint with David Pollack). David Pollack and Robert Pollack have begun to do computations that will give approximations to $H^*(D)$ and hopefully enable us to say something definite about the structure of an eigenvariety.