Lectures by Berger and Breuil on aspects of the p-adic local Langlands' program

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1. Introduction

Berger and Breuil will present in these lectures some on-going research in the construction of p-adic local Langlands correspondence.

As an instance of this correspondence is the following: given a 2-dimensional p-adic crystalline representation V of the local Galois group $G_{\mathbb{Q}_p} = \mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$, there's a construction which attach to V a p-adic (unitary) Banach representation $\Pi(V)$ of the group $\mathrm{GL}_2(\mathbb{Q}_p)$. It turns out that the theory of (φ, Γ)-modules is of crucial use is the analysis of the representation $\Pi(V)$.

Berger will give two lectures on "Galois representations and (φ, Γ)-modules". In these two lectures, detailed background on (φ, Γ)-modules will be given, and they are used to construct representations of the Borel subgroup of $\mathrm{GL}_2(\mathbb{Q}_p)$.

The goal of Breuil's four lectures is to give four points of view on the p-adic local Langlands programme, the basic link between these four points of view being the quest for "interesting" p-adic (i.e. Banach or locally analytic) or mod p representations of $\mathrm{GL}_n(L)$ where L is of a finite extension of $\mathbb{Q}_p$. The first lecture will focus on the basic question: when does a locally algebraic representation of $\mathrm{GL}_n(L)$ admit at least one invariant norm? The second lecture will focus on (φ, Γ)-modules (and the group $\mathrm{GL}_2(\mathbb{Q}_p)$). The third lecture will focus on Drinfeld spaces (also for the group $\mathrm{GL}_2(\mathbb{Q}_p)$). The last lecture will focus on mod p representations (for $\mathrm{GL}_2(L)$).

References

[1] L.Berger, *An introduction to the theory of p-adic representations*, Geometric Aspects of Dwork Theory, 255-292 (2004), also available on Berger's webpage.

[2] L.Berger, C.Breuil, *Towards a p-adic Langlands programme*, available on Berger's or Breuil's webpage.