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The Dirichlet Problem For Fully Nonlinear Equations on Riemannian Manifolds

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Abstract: Manifolds with geometric structure carry large and useful families of non-standard "subharmonic" functions. For example, any almost complex manifold with hermitian metric carries plurisubharmonic functions. Moreover, it also carries "Lagrangian subharmonic functions" whose restrictions to Lagrangian submanifolds are subharmonic. Similarly, a manifold with calibration φ carries "φ-plurisubharmonic functions" which are subharmonic on all calibrated submanifolds.

In all cases the extremals in these families, the "harmonic functions", are interesting and often satisfy a basic non-linear second-order equation.

I will discuss the Dirichlet Problem for such harmonic functions on bounded domains in a riemannian manifold. Existence and uniqueness will be established for quite general second-order equations. The result holds for all continuous boundary data subject to a geometric *F-convexity* of the boundary, defined entirely in terms of the equation F.

Examples include all branches of the Monge-Ampère equation over **R**, **C** and **H**, and all branches of the special lagrangian potential equation.