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On Degenerate Elliptic Monge-Ampere Equations

Jiaxing Hong
School of Mathematical Science, Fudan University
Shanghai, 200433, China
email: jxhong@fudan.edu.cn.

Abstract: Consider a kind of degenerate elliptic Monge-Ampere equations

$$\det(D^2u) = K(x)f(x, u, Du) \text{ in } \Omega \subset R^2 \text{ with } u = 0 \text{ on } \partial\Omega$$

Suppose that $f \in C^\infty$ is positive and $\Omega \in C^\infty$ convex and that $K = d^m\tilde{K}$ for some integer m and smooth positive function $\tilde{K}$ where d is the defining function of $\partial\Omega$. Then we have

Theorem 0.1 *Any C^2-solution to the above problem is in $C^\infty(\bar{\Omega})$. Moreover, if f satisfies some natural structure condition, the above problem always admits a unique solution smooth up to the boundary*

As an application

Theorem 0.2 *The eigenvalue problem*

$$\det(D^2u) = \lambda u^2 \textit{ in } \Omega \subset R^2 \textit{ with } u = 0 \textit{ on } \partial\Omega$$

always has a solution (λ, u) where $u \in C^\infty(\bar{\Omega})$ and convex provided that Ω is smooth convex

Our arguments consists of two main ingredients. One is to give a positive lower bound for Δu and another is to present a priori estimates for a class of linear degenerate elliptic problem which is very closely related to the above degenerate elliptic Monge-Ampere equations.