Conference on Differential Geometry. Calabi-Yau

A conference in celebration of the 70th Birthday of Shing-Tung Yau

Conference Program

Thursday, May 2

8:35 - 8:50 am Morning Refreshments

8:50-9:00 AM
Welcoming Remarks

9:00-10:00 AM Simon Donaldson, Imperial College
Title: "Exceptional Holonomy and Calabi-Yau Geometry"

Abstract: There are many interesting differential-geometric questions involving Riemannian manifolds of dimension 7 and 8 with exceptional holonomy, but for the most part general answers to these questions seem far off. The theme of this talk is the fact that most of what is known about these exceptional structures is derived from strong information in complex differential geometry provided by Yau's solution of the Calabi conjecture, and related results. We will discuss a number of topics within this framework involving both older and more recent developments. These include the "twisted connected sum" construction developed by Kovalev and others, analogous constructions in Yang-Mills theory (Sa Earp and others) and an adiabatic description of associative submanifolds (work in progress with C. Scaduto).

10:00-11:00 AM Kefeng Liu, The University of California, Los Angeles
Title: "A Global Method in Solving Equations on Manifolds"

Abstract: Motivated by the local integral operators introduced by Ahlfors and others in solving the Beltrami equations on discs, we introduce corresponding global operators on manifolds to study variations of complex structures over Kahler manifolds. Furthermore, by using these operators from Hodge theory on a complete Kahler manifold, we construct closed formulas for holomorphic canonical forms under global variations of complex structures and on their moduli spaces, we also present several applications including a simple method solving the general Belrtami equations, and a revisit of the Kuranishi obstruction equations in deformation of complex structures.

11:00 - 11:20 am Coffee Break

11:20-12:20 PM Jun Li, Stanford University
Title: "All genus GW Invariants of Quantic Calabi-Yau Threefold"

Abstract: We will report the recent progress in using Mixed-Spin-P fields to prove Yamaguchi-Yau polynomiality and Bershadsky–Cecotti–Ooguri–Vafa Feynman diagram sum conjectures for GW invariants of quantic Calabi-Yau threefold. This is a joint work with Huailiang Chang and Shuai Guo.

12:20 - 1:40 pm Lunch

1:40-2:40 PM Chiu-Chu Melissa Liu, Columbia University
Title: "Mirror Dymmetry for Formal Toric Calabi-Yau Manifolds"

Abstract: In "A mathematical theory of the topological vertex", Jun Li, Kefeng Liu, Jian Zhou, and the speaker introduced the notion of formal toric Calabi-Yau graphs. A formal toric Calabi-Yau graph determines a formal toric Calabi-Yau threefolds whose Gromov-Witten invariants can be computed by the algorithm of the topological vertex. In this talk, I will describe a version of mirror symmetry for certain formal toric Calabi-Yau threefolds.

2:40-3:40 PM NIKE SUN, THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Title: "On the Ising perceptron model"

Abstract: The perceptron is a toy model of a simple neural network that stores a collection of given patterns. Its analysis reduces to a simple problem in high-dimensional geometry, namely, the intersection of the cube (or sphere) with a collection of random half-spaces. Despite the simplicity of this model, its high-dimensional asymptotics are not very well understood, particularly in the case of the cube. I will describe what is known and present recent results. This lecture is based on joint work with Jian Ding.

3:40 - 4:00 pm Coffee Break

4:00-5:00 PM VALENTINO TOSATTI, NORTHWESTERN UNIVERSITY

Title: "Collapsing Calabi-Yau Manifolds"

Abstract: I will discuss the problem of understanding the limits of Ricci-flat Kahler metrics on compact Calabi-Yau manifolds as the Kahler class or the complex structure degenerate, focusing on the delicate case when collapsing occurs (the limiting space is of lower dimension), which is the situation relevant for the Strominger-Yau-Zaslow picture of mirror symmetry. This is based on joint works with Hein and Zhang.

5:00 - 7:00 pm Reception, The Austine & Chilton McDonnell Common Room, Science Center, 4th Floor

FRIDAY, MAY 3

8:45 - 9:00 am Morning Refreshments

9:00-10:00 AM RICHARD SCHOEN, THE UNIVERSITY OF CALIFORNIA, IRVINE

Title: "A Survey of Some Recent Progress in Mathematical Relativity"

Abstract: This talk will discuss the marginally outer trapped surface (MOTS) equation and the related Jang equation. This equation was originally used by S. T. Yau and the speaker to prove the spacetime positive energy theorem. We then used it to prove that large concentrations of matter force the formation of trapped surfaces and hence the spacetime is singular. It has also been used to construct MOTS under suitable boundary conditions. In this talk we will give the history of this equation, summarize its applications, and discuss current issues which may be related to this approach.

10:00-11:00 AM FAN CHUNG GRAHAM, THE UNIVERSITY OF CALIFORNIA, SAN DIEGO

Title: "Geometric Aspects in Spectral Graph Theory"

Abstract: We will briefly survey some recent developments and open problems in spectral graph theory. In particular, we will discuss the the geometric implications of the spectral gaps including the expansion of vertex/edge neighborhoods and the edge stretches in eigenfunctions using the curvature of graphs.

11:00 - 11:20 am Coffee Break

11:20-12:20 PM CHUU-LIAN TERNG, THE UNIVERSITY OF CALIFORNIA, IRVINE

Title: "Integrable Curve Flows and KdV Type Equations"

Abstract: I will discuss two third-order integrable curve flows in this talk. One is the non-linear Airy flow on Euclidean n-space; this is the curve flow having the normal covariant derivative of the mean curvature vector field of the curve as velocity. The principal curvatures of this curve-flow evolve according to the vector modified KdV equation. The second is the affine curve-flow, a third order curve-flow on the affine n-space that is invariant under the action of SL(n,R) and preserves the volume element. The affine curvatures of these flows evolve according to a Gelfand-Dickey flow (when n=2, this is the KdV). We use techniques in soliton theory to study the Cauchy problems and Hamiltonian aspects of these flows. We also construct infinitely many commuting curve flows, Darboux transforms that generate new solutions from a given one, and explicit soliton solutions. I will first give a careful discussion of these results for the case n=2, then for general n as time permits.

12:20 - 1:40 pm Lunch

1:40-2:40 PM NIGEL HITCHIN, OXFORD UNIVERSITY

Title: "ALE Spaces Revisited"

Abstract: Kronheimer's construction of ALE spaces as hyperkähler quotients is very natural and beautiful but it is difficult to produce concrete formulas from it. In the case where there is a circle action we consider how to obtain the metric on a fixed point set, a rational curve which is a component of the resolution of the ADE singularity. The approach makes use of Penrose's twistor theory and the Slodowy slice.

2:40-3:40 PM TRISTAN COLLINS, THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Title: "Some Results in Stominger-Yau-Zaslow Mirror Symmetry"

Abstract: Mirror symmetry originally arose as a mysterious duality between Calabi-Yau threefolds, interchanging complex and symplectic structures. This duality has since expanded to include a much broader collection of objects, including Fano manifolds, and Landau-Ginzburg models. Two fundamental themes in mirror symmetry are (1) the existence of special Lagrangian fibrations, as conjectured by Strominger-Yau-Zaslow and (2) the correspondence between "stable" objects as predicted in work of Thomas-Yau, and Douglas. Here, stable objects are meant to be special Lagrangian manifolds on the symplectic side, and holomorphic bundles with canonical metrics, on the complex side. I will report on recent results in both of these directions. This talk will discuss joint works with A. Jacob, Y.-S. Lin, and S.-T. Yau.

3:40 - 4:00 pm Coffee Break

4:00-5:00 PM STEPHEN YAU, TSINGHUA UNIVERSITY, P.R. CHINA

Title: "Torelli Theorem for k-th Yau Algebras over Simple Elliptic Singularities E6~"

Abstract: The well known Mather-Yau Theorem states that the complex structure of an isolated hypersurface singularity determines and is determined by its moduli algebra. This gives a natural connection between the theory of isolated hypersurface singularities and the theory of finite dimensional commutative local algebras. Later Yau introduced a natural Lie algebra attached to the moduli algebra. He showed that this Lie algebra is solvable. It is a natural question to ask whether the map from {isolated hypersurface singularities of fixed dimension} to {finite dimensional solvable/nilpotent Lie algebras} is injective. Geometrically, this means whether the Lie algebra structure varies when the complex structure of the singularity varies. These Lie algebras are called Yau algebras. Almost 30 years ago, Seeley and Yau proved that the answer of the above question is positive for both E7~ and E8~ families of singularities. However Yau showed that the Lie algebra of the family E6~ is constant. In this way he discovered the Yau algebra of E6~ has a natural continuous family of nonequivalent finite dimensional representation. For the last thirty years, it has been mysterious to people why such phenomenon exits. In this talk, we consider the k-th moduli algebra and its associated k-th Yau algebra. The zeroth moduli algebra and zeroth Yau algebra are just the classical moduli algebra and Yau algebra respectively. Since there are a series of finite dimensional solvable Lie algebras attached to an isolated hypersurface singularity, it is natural to conjecture that the Torelli type theorem holds. We focus on the singularity E6~ and prove a Torelli type theorem for k-th Yau algebra for all $k > 1$. Moreover, we show that the first Yau algebra is constant, which provides a new example of continuous family of nonequivalent finite dimensional representation. At last, we develop an algebraic method to characterize the set of isomorphisms of both k-th moduli algebra and k-th Yau algebra. This is a joint work with C. Q. Hu and H. Q. Zuo.

SATURDAY, MAY 4

8:45 - 9:00 am Morning Refreshments

9:00-10:00 AM KAREN UHLENBECK, UNIVERSITY OF TEXAS

Title: "Morrey Spaces and Regularity for Yang-Mills-Higgs Equations"

Abstract: We start with background on regularity theory for the equations of gauge theory. Morrey spaces arise naturally from monotonicity theorems in dimensions greater than 4, and singular sets of Hausdorff codimension 4 arise in limiting processes. Our main technical result is that functions in a Morrey space which satisfy an elliptic inequality off a singular set of Hausdorf codimension 4 can be bounded in the interior in a much better Morrey space. This can be used to show that: when the curvature is bounded by the sum of a smooth function and a term small in the relevant Morrey space, solutions of a Yang-Mills-Higgs equation off a singular set of codimension 4 are gauge equivalent to smooth solutions in the interior. This work, which is joint with Penny Smith, simplifies and extends work of Tao-Tian and Meyer-Riviere.

10:00-11:00 PM DUONG H. PHONG, COLUMBIA UNIVERSITY

Title: "On Some New Curvature Flows in non-Kähler Geometry"

Abstract: The 2006 proof by J.X. Fu and S.T. Yau of the existence of a non-Kähler solution to the Hull-Strominger system has pointed to the compactifications of string theories as the natural arena for a

continuation of Yau's earlier landmark works on the Calabi conjecture and canonical metrics. A major problem for such continuations in non-Kähler geometry is the absence of a $\partial\bar{\partial}$ lemma. We propose "anomaly flows" as a systematic way of bypassing this difficulty. We show how this method can recover Yau's solution of the Calabi conjecture as well as the Fu-Yau solution of the Hull-Strominger system. We also discuss the many new analytic difficulties which arise, and which may be interesting in their own right, from the point of view of the theory of non-linear partial differential equations. This is joint work with Teng Fei, S. Picard, and X.W. Zhang.

11:00 - 11:20 am Coffee Break

11:20-12:20 PM ALINA MARIAN, NORTHEASTERN UNIVERSITY

Title: "Universal Integrals over Hilbert Schemes of Points"

Abstract: The calculation of intersection-theoretic invariants for Hilbert schemes of points on a smooth surface is a basic problem in enumerative geometry. One fundamental series of invariants is given by the top Segre classes of tautological vector bundles over the Hilbert scheme. Another one, the Verlinde series, encodes holomorphic Euler characteristics of line bundles on the Hilbert scheme. The two are conjecturally related. I will describe this circle of ideas, as well as recent progress evaluating Segre integrals, in a joint project with D. Oprea and R. Pandharipande.

12:20 - 1:40 pm Lunch

1:40-2:40 PM CLIFFORD TAUBES, HARVARD UNIVERSITY

Title: "Observations on Compactness for First Order Yang-Mills-Higgs like Equations in Dimensions 3 and 4"

Abstract: The talk will concern the behavior of non-convergent sequences of solutions to first order, gauge theory equations in dimension 4 (and 3) that are generalizations of the anti-self-dual Yang-Mills equations and the Seiberg-Witten equations. (For example, multi-spinor Seiberg-Witten equations, the Vafa-Witten equations and Kapustin-Witten equations.). The limiting behavior leads to some new geometric objects, what are called now Z/2 harmonic differential forms and spinors. I'll explain what these are and what we know about them (which shouldn't take more than a few minutes since we know very little about them). I hope also to talk about some special cases of the Kapustin-Witten equations (relevant to Witten's conjecture about the Jones polynomial for knots in S^3) where Z/2 harmonic objects won't appear, but other interesting geometry does.

2:40-3:40 PM LYDIA BIERI, UNIVERSITY OF MICHIGAN

Title: "The Einstein Equations and Radiation from a Geometric-Analytic Point of View"

Abstract: The Einstein equations describe the laws of the universe and lie at the core of General Relativity (GR). Their very nature is geometric, they can be written as a system of nonlinear hyperbolic partial differential equations. Mathematical General Relativity has flourished through the input from Geometric Analysis in the last decades. In a creative way, these fields have fueled each other resulting in new methods leading to major breakthroughs. In this talk, I will present geometric-analytic investigations answering questions about gravitational waves. These are fluctuations of the curvature of the spacetime. Typically, they

are produced during the mergers of black holes or neutron stars and in core-collapse supernovae. In 2015, gravitational waves were observed for the first time by Advanced LIGO (and several times since then). Understanding gravitational radiation is tightly interwoven with the study of the Cauchy problem in GR. I will also discuss the memory effect of gravitational waves, a permanent change of the spacetime. With P. Chen and S.-T. Yau we showed that a component of the electromagnetic field in the Einstein-Maxwell equations contributes to this memory. I will also address my latest work on the nature of gravitational radiation and memory for various classes of spacetimes.

3:40 - 4:00 pm Coffee Break

4:00-5:00 PM MU TAO WANG, COLUMBIA UNIVERSITY

Title: "Gravitational Radiation and Quasi-local Mass"

Abstract: An observer of an astronomical event is situated at future null infinity, where light rays emitted from the source approach. Mathematically, null infinity corresponds to the portion of the spacetime boundary defined by equivalence classes of null geodesics. In the talk, I shall discuss new applications of the theory of quasilocal mass and quasilocal conserved quantities to the study of future null infinity. This is based on joint work with Po-Ning Chen, Jordan Keller, Ye-Kai Wang, and Shing-Tung Yau.

SUNDAY, MAY 5

8:45 - 9:00 am Morning Refreshments

9:00-10:00 AM EDWARD WITTEN, INSTITUTE FOR ADVANCED STUDY

Title: "Unorientable Two-Manifolds, Super Riemann Surfaces, And Random Matrices"

Abstract: Recently, P. Saad, S. Shenker, and D. Stanford showed that what is arguably the simplest model of quantum gravity, which is the Jackiw-Teitelboim (JT) model in two spacetime dimensions, can be understood as a random matrix theory. This result depends on the facts that JT gravity computes volumes of moduli spaces, and on the fact that those volumes have a random matrix interpretation. In this talk (reporting on work with D. Stanford), I will explain how to extend these results to the case that an ordinary Riemann surface is replaced by an unorientable two-manifold and/or a super Riemann surface.

10:00-11:00 AM CUMRUN VAFA, HARVARD UNIVERSITY

Title: "Towards Completion of Curve Counting for Calabi-Yau 3-folds"

Abstract: In this talk I review the progress made in computing topological string partition function on compact Calabi-Yau manifolds.

11:00 - 11:20 am Coffee Break

11:20-12:20 PM ANDREW STROMINGER, HARVARD UNIVERSITY

Title: "Symmetries, Memory and the Scattering Problem in General Relativity"

Abstract: The scattering problem in general relativity concerns the evolution of initial data at past null infinity to final data at future null infinity. This differs from the more commonly considered initial data problem on spacelike slices in that non-trivially acting diffeomorphisms, such as the supertranslations of Bondi, Metzner, van der Burgh and Sachs or the superrotations of Barnich and Trossaert, play a central role.
ecent progress on specifying the space of initial data, the infinite number of conservation laws and the surprising connection to two dimensional conformal field theory are described.