

# Current Developments in Mathematics 1996

*Edited by*

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Current Developments in Mathematics, 1996

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## Preface

These are the proceedings of the joint seminar by M.I.T. and Harvard on the current developments in mathematics for the year 1996. Established in 1995, this seminar will be continued each year. In 1997 and following years, we will also add a section on open problems.

The organizing committee for the seminar is made up of representatives from the mathematics departments of the two institutions: Raoul Bott, Arthur Jaffe, and S.T. Yau from Harvard; and David Jerison, George Lusztig, and Isadore Singer from M.I.T.

We would like to thank each of the contributors and to recognize the several institutions without whose participation the seminar would not have been possible: the departments of mathematics at Harvard and MIT for their financial aid and the American Academy of Arts and Sciences for opening their facilities to us.

*Editors*



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