

Current Developments in Mathematics 1995

Edited by

Raoul Bott
Michael Hopkins
Arthur Jaffe
Isadore Singer
Daniel W. Stroock
Shing-Tung Yau

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Editorial Board:

Raoul Bott
Arthur Jaffe
Shing-Tung Yau

Harvard University
Cambridge, Massachusetts

Michael Hopkins
Isadore Singer
Daniel W. Stroock

Massachusetts Institute of Technology
Cambridge, Massachusetts

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Preface

In the fall of 1994, the departments of mathematics of Harvard University and the Massachusetts Institute of Technology joined forces to establish what they hope will be an annual seminar devoted to the exposition of current developments in mathematics. Each year the content of the seminar will be determined by a committee made up of representatives from each of the two departments. For the 1995 seminar, the members of the committee were Raoul Bott, Arthur Jaffe, and Shing-Tung Yau from Harvard, and Michael Hopkins, Isadore Singer and Daniel Stroock from M.I.T.

Although the committee attempts to choose subjects which they feel are of current interest, time constraints make it inevitable that only some important subjects can be covered. For this reason, the committee will sometimes avoid subjects which already enjoy wide exposure in seminars elsewhere. For example, this year the committee decided not to have talks about gauge theory invariants, in spite of the obvious appeal and enormous progress in this field.

Because the 1995 seminar was not even conceived until December 1994 and the speakers were not notified until the turn of the year, it is remarkable that we were able to produce the preliminary volume from manuscripts which were delivered in early April 1995, and, for this, we are all in the authors' debt. We hope that their efforts will be rewarded by the feedback which their talks and manuscripts provoke.

Finally, the organizers take this opportunity to thank several institutions without whose cooperation the seminar would not have been possible: the departments of mathematics at Harvard and M.I.T. for financial aid and the American Academy of Arts and Sciences for opening their facilities to us.

Editors

Contents

Fermat's last theorem by H. Darmon, F. Diamond and R. Taylor		1
1	Elliptic curves and modular forms	15
2	Galois theory	45
3	Modular forms and Galois representations	77
4	Hecke algebras	97
5	Commutative algebra	125
Renormalization ideas in conformal dynamics by M. Lyubich		155
1	Introduction	156
2	Combinatorics of complex unimodal maps	158
3	Parameter plane vs dynamical plane	163
4	Geometric bounds	165
5	Rigidity	172
6	Universality	179
7	Measure of Julia sets	183
8	Appendix: Quasi-conformal maps	184
Algebraic K-theory and traces by Ib Madsen		191
1	Introduction	192
2	Topological cyclic homology	199
3	The relative theorems	229
4	The absolute theorems	258
5	Calculations in K -theory	286
The classification of conformal dynamical systems by C.T. McMullen		323
1	Introduction	323
2	Finiteness theorems and Teichmüller space	325
3	Expanding dynamics	332
4	Topology of hyperbolic 3-manifolds	335
5	Topology of rational maps	340
6	Renormalization	347
7	Notes and references.	352

Quantum cohomology and its associativity by <i>G. Tian</i>		361
1	Introduction	361
2	The GW-invariants	364
3	More Examples and Applications	370
4	The Composition Law	374
5	Quantum Cohomology	376
6	The WDVV Equation	381
7	Applications to Enumerative Geometry	389
8	The Generalized Witten Conjecture	390
9	Quantum Cohomology and Symplectic Floer Cohomology . .	392
10	Degenerating Families	395
11	Mirror Symmetry and Calabi-Yau Manifolds	397